UDC 551.587:551.557.31:551.547.31:551.524.731(968.1)

DIURNAL VARIATION OF WIND, PRESSURE, AND TEMPERATURE IN THE TROPOSPHERE AND STRATOSPHERE OVER ENIWETOK

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ABSTRACT

The diurnal and semidiurnal variations of wind, pressure, and temperature at 23 levels between 1000 and 10 mb over Eniwetok are obtained by combining two series of 6-hr soundings taken at different times in April–July 1956 and 1958. The prevalence of the semidiurnal pressure wave in the lower layers gradually yields to the first harmonic in the upper troposphere and stratosphere, as a consequence of the predominantly diurnal variation of temperature. The semidiurnal variation of temperature is found to increase slightly with height. Increase of the second harmonic amplitude of the u and v wind components with height is roughly proportional to the decrease of air density, with the phase remaining nearly constant, except for the lower layers. The phase relationship between the semidiurnal variations of pressure, and eastward and northward directed wind components is discussed in terms of a model based on a linearized form of the equations of motion, frictionless flow, and the assumption that the oscillations are simple progressive waves.

1. INTRODUCTION

Periodic variations in the daily march of atmospheric pressure are particulaly obvious in the Tropics and have been known for centuries. Theoretical studies on atmospheric tides (for example, Bartels 1928, Wilkes 1949, Stolov 1955, Siebert 1961) have also been interested in the periodic wind variations that are associated with the observed solar diurnal, semidiurnal, and terdiurnal oscillations of surface pressure. While the periodic pressure variations are readily apparent even on the barograph trace of individual days, an empirical study of tidal wind oscillations calls for a longer period of observations.

Hann (1902, 1908, 1909, 1910) and Gold (1910) seem to have been the first to present impressive observational evidence of tidal variations in the surface winds. Hann evaluated records of varying length from several mountain stations in the Tropics and in temperate latitudes, and Gold analyzed 16 yr (1892–1907) of hourly data at the Island of St. Helena in the South Atlantic. Continuous wind measurements aboard ship were performed during the Meteor Expedition of 1925–1927 in various regions of the tropical North and South Atlantic. Kuhlbrodt and Reger (1938) subjected this body of data to a careful statistical analysis and were thus able to demonstrate the existence of a marked semidiurnal variation in the zonal and meridional components of the surface wind. This was further corroborated by Bartrum's analysis (1957) of the daily march of surface wind and pressure at Bermuda in 1935–1936.

The study of periodic variations in the upper air winds is hampered by the fact that operational soundings are as a rule made twice, or at most four times daily. Fletcher (1959) bypassed this difficulty by using soundings taken during 1948–1954 at 0300, 0900, 1500, and 2100 GMT at Guam and Bermuda, that is, at stations in different latitudes and longitudes, and combining them according to local time. Study of these data was continued by Rudloff (1966). Harris (1959) in his analysis of upper air wind variations over Washington, D.C., took advantage of the change in the sounding schedule, from 0300, 0900, 1500, and 2100 GMT to 0000, 0600, 1200, and 1800 GMT, which occurred on June 1, 1957. He thus combined the observations of June, July, and August of 1956 and 1957 into a single series. Similarly, a change in the sounding schedule at Lajes Field, Terceira, Azores, on Apr. 1, 1957, enabled Harris et al. (1962) to combine the series April 1956 through March 1957 with April 1957 through March 1958 to obtain eight upper air soundings per day.

Furthering Riehl's evaluation (1947) and Haurwitz' evaluation (1947) of a 2-mo period of 3-hr upper air soundings in the eastern Caribbean, the studies of Harris (1959) and Harris et al. (1962) also included an analysis of the diurnal march of upper air pressure and temperature.

The present study is based on data collected during special upper air programs over the tropical Pacific. Soundings taken during different periods are combined in a way similar to that used in earlier empirical investigations of atmospheric tidal oscillations.

2. OBSERVATIONAL DATA

Unusually intense upper air programs have been operated over the west-central Pacific in connection with the various nuclear tests (Joint Task Force Seven Meteorolog-

TABLE 1.—*Number of soundings at given constant pressure levels*

Level (mb)	Apr.-July 1956				Apr.-July 1958			
	GMT 03	09	15	21	GMT 00	06	12	18
Sfc.	113	98	114	100	120	101	121	106
1000	110	96	113	100	118	100	116	106
950	110	96	113	100	118	100	116	106
900	110	96	113	100	118	99	115	106
850	111	96	113	100	120	99	117	105
800	112	96	113	100	120	99	118	105
750	112	97	113	100	120	100	119	106
700	112	97	114	100	120	100	120	106
650	112	97	114	99	120	100	120	106
600	112	97	114	100	120	101	121	106
550	112	97	113	100	120	101	121	106
500	112	96	113	100	120	101	121	106
450	112	96	113	100	120	100	121	106
400	112	94	113	100	120	101	120	106
350	112	96	113	100	119	101	120	106
300	112	96	112	100	118	101	118	106
250	112	96	112	100	118	100	118	105
200	112	95	112	98	119	99	119	105
150	111	94	109	97	119	99	118	105
100	104	91	100	90	119	98	117	101
50	101	82	91	86	114	90	107	99
30	98	77	81	84	109	87	98	89
20	95	70	71	77	103	75	86	75
10	50	26	23	45	71	61	13	11

ical Center 1958, 1960). Of these, the meteorological data collections of operations Redwing and Hardtack in the summers of 1956 and 1958, respectively, were found to be most useful for the purposes of the present study. Thermodynamic data and winds for constant pressure levels for April–July of 1956 and 1958 were obtained from the National Weather Records Center at Asheville, N.C., on magnetic tape. Wind direction is reported to the nearest degree, and wind speed in tenths of m sec^{-1}. Height of constant pressure levels is given in full gpm, temperature in tenths of °C, and relative humidity in percent.

During April–July 1956, standard sounding hours were 0300, 0900, 1500, and 2100 GMT, and in April–July 1958, standard hours were 0000, 0600, 1200, and 1800 GMT. During both periods, numerous additional soundings were made at other than the standard hours, in 3-hr intervals. Only observations taken at the respective standard hours were used here. From the dense network operated in the Marshall Islands area during both Redwing and Hardtack, Eniwetok (11°21′ N., 162°20′ E.) was chosen for initial analysis, since it is one of the stations with most nearly complete data coverage. The number of the upper air observations is summarized in table 1; constant pressure levels in a 50-mb spacing between 1000 and 50 mb were used, as well as the surface and the 30-, 20-, and 10-mb surface levels.

The observing site is located on the southern shore (adjacent to the aircraft runway) of Eniwetok Island near its southwestern tip. The meteorological exposure is considered to be excellent and is unobstructed from all directions. Eniwetok Island is a sand-covered coral reef with a maximum elevation above sea level of about 20 ft. The island has been cleared of all vegetation except for widely scattered coco palms and scrub at the northeastern end (Joint Task Force Seven Meteorological Center 1956).

3. COMPUTATIONAL PROCEDURES

Elements processed included west-east and south-north components of wind, u and v, temperature, T, and height of constant pressure levels, h. For each of these elements and the levels listed in table 1, monthly means were computed for the standard sounding hours, that is 0300, 0900, 1500, and 2100 GMT for April, May, June, and July of 1956, and 0000, 0600, 1200, and 1800 GMT for the same months of 1958.

Deviations of each standard hour from the 4-hr average were computed next. These deviations from the respective monthly mean u', v', T', and h' (for brevity, henceforth referred to as zonal and meridional wind components, u and v, temperature T, and height of constant pressure surfaces, h), were then combined for 1956 and 1958, so as to yield a new time series of eight 3-hr observations for each month.

This procedure of combining two series of four observations each can be justified if the daily march can be satisfactorily represented by the first two terms of a Fourier series. For details, reference is made to Harris (1959) and Johnson (1955).

The monthly series of eight 3-hr data thus obtained was subjected to harmonic analysis.

If $Y(t)$ is the time series being analyzed, the resulting harmonic components can be expressed as

$$Y_\nu(t) = C_\nu \cos \nu(\theta - \alpha_\nu) \tag{1}$$

Here, θ denotes the hour angle, ν the number of the harmonic, and α_ν corresponds to the time at which the νth harmonic has a maximum, being counted from an origin of 0000 LT. For details, reference is made to standard texts (for example, Panofsky and Brier 1965). The term "daily variation" will here be used to denote the combination of the diurnal and semidiurnal oscillations.

It should be recalled that Harris (1959), Harris et al. (1962), Haurwitz (1947), and others used a sine function instead of equation (1). Their phase angles can thus not be directly compared. For inclusion in figures 1 through 4 and table 3, phase angles were converted to conform with equation (1). For example, in the studies of Harris and Harris et al., the time of the maximum is given by $t_{max} = (450° - \alpha_\nu)/15$, whereas the time of the maximum according to equation (1) is $t_{max} = \alpha_\nu/15$. Concerning the phase angles in figures 1 through 4 and tables 2 through 5, $\alpha_\nu/15$ yields the time of the maximum in hours local time.

Fourier coefficients P_ν, Q_ν, of the 4 mo were arithmetically averaged, so as to yield amplitude and phase angle in equation (1), representative of the entire season according to local time.

TABLE 2.—*Diurnal and semidiurnal variation of pressure in April–July 1956 and 1958 at Eniwetok. Amplitude, A, is in millibars; and phase, α, in degrees counted from local midnight. The percent symbol, %, denotes percentage variance explained by a particular harmonic. "Observed" refers to values obtained from pressure observations, and "Computed" to values derived from wind data according to equation (8).*

Level (mb)	Diurnal					Semidiurnal				
	Observed			Computed		Observed			Computed	
	A	α	%	A	α	A	α	%	A	α
1000	0.35	114	10	1.51	51	1.04	182	90	0.34	307
950	.41	117	11	1.41	32	1.20	182	89	.20	328
900	.27	126	7	1.80	20	0.98	182	93	.81	319
850	.22	132	6	1.30	353	.92	181	94	.93	318
800	.21	137	5	1.83	17	.89	181	94	1.03	299
750	.19	152	5	1.87	32	.79	178	94	1.28	329
700	.18	154	5	1.51	43	.78	181	94	1.36	329
650	.23	169	7	2.13	43	.84	179	93	0.94	313
600	.19	176	7	1.47	68	.68	180	91	.70	332
550	.23	170	12	1.21	38	.62	178	86	.89	337
500	.21	188	12	2.23	35	.58	181	86	.55	166
450	.27	193	22	1.25	18	.50	181	75	1.38	320
400	.27	196	22	0.92	23	.49	181	75	1.07	329
350	.30	202	30	.63	350	.45	182	68	1.23	165
300	.32	205	40	.61	41	.39	182	58	1.35	323
250	.33	205	48	.38	0	.35	182	51	1.15	339
200	.36	206	63	.39	230	.27	182	36	0.64	166
150	.36	207	76	.28	255	.20	181	23	.51	165
100	.32	198	84	.62	171	.13	177	14	.46	172
50	.23	189	81	.23	117	.09	182	14	.23	341
30	.16	176	89	.02	180	.05	183	8	.19	181
20	.14	174	95	.05	48	.03	189	3	.15	176
10	.13	183	91	.20	42	.02	173	3	.01	177

Amplitudes of the height variations of constant pressure surfaces were converted into amplitudes of pressure (p) variations, using the hydrostatic equation and the mean summer atmosphere over Eniwetok, as obtained from the April–July 1956 and 1958 soundings.

In addition to the direct harmonic analysis of the height of constant pressure surfaces, h, Fourier coefficients for the first and second harmonics of h were also computed from the corresponding coefficients for u and v, following the theory presented in section 7.

The percentage variance explained by each harmonic was computed as described in standard texts (for example, Panofsky and Brier 1965).

4. PRESSURE

Results of the harmonic analysis of pressure surface heights are presented in table 2. Amplitude and phase of the diurnal and semidiurnal pressure variation are also plotted in figure 1. Harmonics of height variation as computed from winds (section 7) are included in table 2. Values obtained by Haurwitz (1947) for the eastern Caribbean, by Harris (1959) for Washington, D.C., and by Harris et al. (1962) for the Azores are included in figure 1 for comparison. Similarly, table 3 summarizes surface data for Balboa, Canal Zone; San Juan, Puerto Rico; Aguadilla, Puerto Rico (Haurwitz and Cowley 1965); Bermuda (Bartrum 1957); and Wake Island (Kiser et al. 1963).

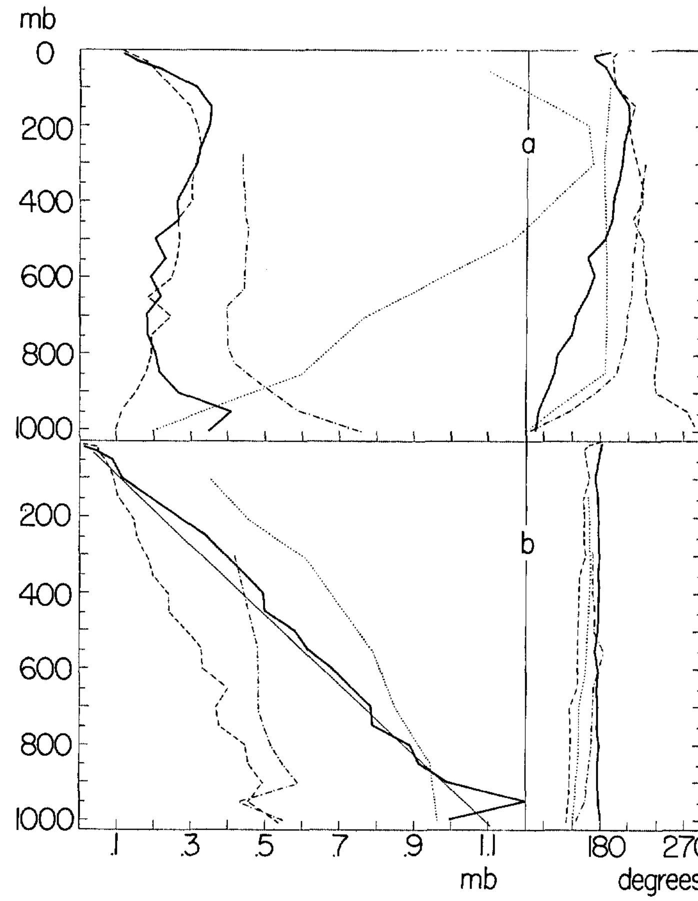

FIGURE 1.—Diurnal (a) and semidiurnal (b) variations of pressure in April–July 1956 and 1958 at Eniwetok (solid lines); Terceira, Azores (broken lines), after Harris et al. (1962); Washington, D.C. (dash-dotted lines), after Harris (1959); and eastern Caribbean (dotted lines), after Haurwitz (1947); amplitude in mb and phase in degrees counted from 0000 LT.

TABLE 3.—*Diurnal and semidiurnal variation of surface pressure at Balboa, Canal Zone (1941–1958), San Juan, Puerto Rico (1945–1962), Aguadilla, Puerto Rico (1941–1950), after Haurwitz and Cowley (1965); at Bermuda (1935–1936) after Bartrum (1957); and at Wake Island (1949–1961) after Kiser et al. (1963). Amplitude, A, is in millibars, and phase, α, in degrees counted from 0000* LT.

Place	Diurnal		Semidiurnal	
	A	α	A	α
Wake	0.29	60	0.77	152
Balboa	.52	81	1.15	151
San Juan	.23	79	0.90	152
Aguadilla	.20	102	.88	153
Bermuda	.19	171	.54	154

The amplitude and phase plots in figure 1 illustrate the importance of the semidiurnal variation in the lower layers, gradually yielding to the dominance of the first harmonic aloft. The daily pressure maxima in the lower layers at Eniwetok occur at approximately 1100 and 2300 LT, with minima at 1700 and 0500 LT, while the maxima

TABLE 4.—*Diurnal and semidiurnal variation of temperature in April–July 1956 and 1958 at Eniwetok. Amplitude, A, is in °C, and phase, α, in degrees counted from 0000* LT. *The percent symbol, %, denotes percentage variance explained by a particular harmonic.*

Level (mb)	Diurnal			Semidiurnal		
	A	α	%	*A*	α	%
Sfc	0.95	210	65	0.51	203	18
1000	.49	223	88	.17	207	10
950	.17	254	71	.06	305	8
900	.18	249	74	.07	317	10
850	.19	251	67	.08	331	12
800	.16	239	78	.03	320	3
750	.15	236	66	.03	302	3
700	.17	217	83	.06	286	9
650	.19	225	78	.02	288	1
600	.19	223	83	.02	204	1
550	.20	207	79	.06	233	7
500	.24	210	82	.06	232	5
450	.25	215	98	.02	213	1
400	.29	224	94	.06	210	4
350	.36	216	93	.02	221	0
300	.31	221	97	.02	333	0
250	.31	218	93	.03	297	1
200	.33	221	96	.06	279	3
150	.40	192	96	.07	297	3
100	.23	172	76	.11	175	18
50	.75	158	96	.05	167	0
30	.66	133	95	.13	251	4
20	1.23	169	91	.25	217	4
10	2.33	165	90	.27	240	1

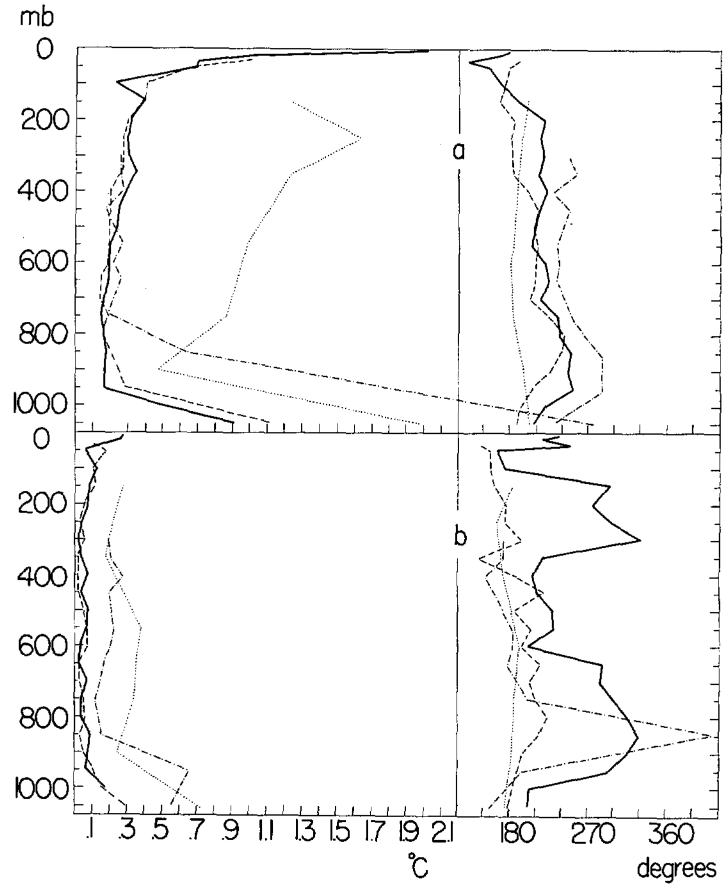

FIGURE 2.—Diurnal (a) and semidiurnal (b) variations of temperature in April–July 1956 and 1958 at Eniwetok (solid lines); Terceira, Azores (broken lines), after Harris et al. (1962); Washington, D.C. (dash-dotted lines), after Harris (1959); and eastern Caribbean (dotted lines), after Haurwitz (1947); amplitude in °C and phase in degrees counted from 0000 LT.

for the second harmonic are computed for approximately 1200 and 2400 (table 2). The phase of the second harmonic in the lower layers is somewhat larger than values obtained in previous studies (for example, Haurwitz 1947, Harris 1959, Harris et al. 1962) and does not exhibit the slight increase with height.

The amplitude of the semidiurnal variation of pressure decreases at about the same rate as pressure itself, as can be seen from the thin straight line entered into figure 1 for comparison. This behavior corresponds to the early findings of Hann (1926) and Wagner (1932) based on mountain observations, as well as the results of Harris et al. (1962) for Terceira, Azores. While expecting this pattern on theoretical grounds, Haurwitz (1947) was unable to verify it in his Caribbean study, based on a relatively short period of upper air soundings. Similarly, Harris (1959) in his analysis based on a 2-mo period of radiosoundings at Washington, D.C., found the second harmonic amplitude of pressure to decrease with height at a rate slower than pressure itself (fig. 1).

When considering the well-known decrease of the second harmonic amplitude of pressure with latitude (for example, Haurwitz and Sepúlveda 1957, Siebert 1961), the magnitude arrived at in the present study compares well with the results from stations summarized in table 3 and figure 1.

In contrast to the semidiurnal variation, the distribution of the first harmonic has been found to be more irregular and subject to pronounced regional characteristics (Wilkes 1949). Rapid phase changes with height were obtained from early mountain observations (Wagner 1932).

The phase of the first harmonic at Eniwetok contrasts particularly strongly with that at Terceira, Azores (fig. 1), but is in closer agreement with the Washington, D.C., and eastern Caribbean data (Harris et al. 1962, Harris 1959, Haurwitz 1947), lacking however the rapid phase change in the lower layers.

As illustrated by figure 1, the amplitudes of the diurnal pressure variation at Eniwetok are in close agreement with the values for Terceira, Azores (Harris et al. 1962). On the contrary, the amplitudes for Washington, D.C. (Harris 1959), are somewhat in agreement, and those for the eastern Caribbean (Haurwitz 1947) are considerably larger.

5. TEMPERATURE

Results of the harmonic analysis of temperature are listed in table 4 and are graphically depicted in figure 2 with values obtained in various previous studies for comparison.

TABLE 5.—*Diurnal and semidiurnal variation of the eastward and northward wind components in April–July 1956 and 1958 at Eniwetok. Amplitude, A, is in m sec^{-1}, and phase, α, in degrees counted from local midnight. The percent symbol, %, denotes percentage variance explained by a particular harmonic.*

Level (mb)	Eastward wind component						Northward wind component					
	Diurnal			Semidiurnal			Diurnal			Semidiurnal		
	A	*α*	%	*A*	*α*	%	*A*	*α*	%	*A*	*α*	%
Sfc.	0.15	230	69	0.09	26	23	0.08	98	9	0.23	166	78
1000	.24	231	49	.16	67	20	.10	140	16	.16	169	42
950	.28	203	78	.02	116	0	.10	215	10	.26	183	60
900	.31	197	86	.12	50	13	.16	127	20	.24	179	45
850	.22	160	55	.17	46	33	.21	119	42	.17	190	28
800	.27	194	53	.19	26	27	.36	113	57	.25	180	26
750	.30	205	56	.23	58	33	.38	135	48	.38	194	46
700	.23	225	37	.28	61	53	.40	129	51	.38	187	46
650	.40	223	73	.19	41	17	.43	133	63	.32	186	33
600	.32	246	69	.16	67	18	.27	164	36	.35	186	61
550	.22	210	43	.21	71	37	.40	140	49	.40	191	50
500	.55	210	88	.14	90	5	.52	138	50	.45	188	38
450	.25	187	30	.41	50	62	.48	124	47	.51	188	52
400	.23	182	27	.36	61	67	.46	141	48	.40	185	36
350	.25	163	18	.47	77	65	.16	106	11	.45	197	85
300	.18	188	8	.63	53	89	.47	164	70	.27	192	24
250	.23	171	9	.64	71	70	.10	187	5	.27	177	35
200	.17	85	13	.44	77	81	.41	283	84	.10	173	5
150	.06	165	1	.44	77	57	.64	332	78	.17	162	6
100	.49	338	48	.51	76	52	.75	282	68	.41	193	21
50	.40	308	35	.54	73	65	.56	189	68	.36	188	28
30	.26	72	6	.83	93	63	.62	179	77	.25	198	13
20	.22	108	4	1.06	90	88	1.29	161	82	.59	177	17
10	2.41	222	79	.18	98	0	1.27	132	77	.40	196	8

Inspection of figure 2 shows that the first harmonic of temperature predominates at all levels. Its amplitude is largest at the surface, decreases to about 950 mb, and increases again gradually into the upper troposphere. Amplitudes of the first harmonic agree closely with the Terceira, Azores, data (Harris et al. 1962), in both the absolute magnitude and the pattern of vertical distribution. Amplitudes at Washington, D.C. (Harris 1959), are considerably larger, particularly in the lower layers. This may reflect the effect of continental location. Diurnal and semidiurnal temperature variations derived for the eastern Caribbean (Haurwitz 1947), however, far exceed those obtained in the later studies, particularly for the middle and upper troposphere.

The diurnal phase of temperature at Eniwetok most closely resembles that at Terceira, Azores, in both magnitude and vertical distribution, while values at Washington, D.C., and in the eastern Caribbean are slightly greater and less, respectively.

The semidiurnal variation exhibits large fluctuations in phase. A slight increase of the second harmonic amplitude with height is indicated in the middle and upper troposphere. As in previous studies, the second harmonic may be considered as being poorly determined. For a discussion of periodic errors in radiosonde temperature measurements, reference is made particularly to Teweles and Finger (1960), Harris et al. (1962), Finger et al. (1964), and Finger and McInturff (1968).

6. WINDS

Results of the harmonic analysis of the u and v wind components are listed in table 5 and are graphically displayed in figures 3 and 4; values for Washington, D.C. (Harris 1959), and Terceira, Azores (Harris et al. 1962), are added for comparison.

The first and second harmonics of the u and v components of wind at Eniwetok are, contrary to the daily march of pressure, of the same general magnitude (fig. 3). As suggested by previous studies (for example, Wallace and Hartranft 1969), the first harmonic is subject to marked regional variations. The daily (first plus second harmonics) variation of total wind increases with height, while air density decreases. The daily variation of the horizontal mass transport would thus remain approximately constant with height, as pointed out by Fletcher (1959).

The first harmonic amplitude of the u component at Eniwetok (fig. 3) is larger than at Terceira, Azores, but smaller than at Washington, D.C., with a similar behavior for the relative magnitude of the v component.

The first harmonic phase at Eniwetok agrees reasonably well with the data for Terceira, Azores; whereas it differs considerably from the Washington, D.C., values below 500 mb, for both the u and v components. The first harmonic phase of v increases with height at both Eniwetok and Terceira up to 100 mb and then rapidly decreases. Contrary to v, the first harmonic phase of u slowly decreases upward, at both Eniwetok and Terceira. For Washington, D.C., no data are available for the upper troposphere.

The second harmonic amplitudes of u and v components at Eniwetok compare reasonably well with those at Terceira, Azores, Washington, D.C., and Bermuda. Agreement is most satisfactory for the u component at Eniwetok and Terceira (fig. 4). Values for both the u and v components in the lower layers at Eniwetok, Terceira, and Bermuda also agree very closely.

The second harmonic phase of both the u and v components at Eniwetok (figs. 3 and 4) agrees strikingly with that at Terceira, Azores (Harris et al. 1962), and at Washington, D.C. (Harris 1959), with no evident height dependency through 10 mb. There is some indication for the phase to be slightly smaller at Eniwetok. The second harmonic phase of u, in particular, shows irregular fluctuations with height at both Eniwetok and Terceira.

The amplitude and phase plots, figures 1, 3, and 4, indicate systematic phase differences between the first and second harmonics of pressure and the u and v wind components. These relationships will be discussed in section 7.

The vertical distribution of amplitude and phase of the wind variation is interesting in relation to some implications of tidal theory which have recently been discussed by Lindzen (1967) and Wallace and Hartranft (1969). As shown by Lindzen, two sets of Hough

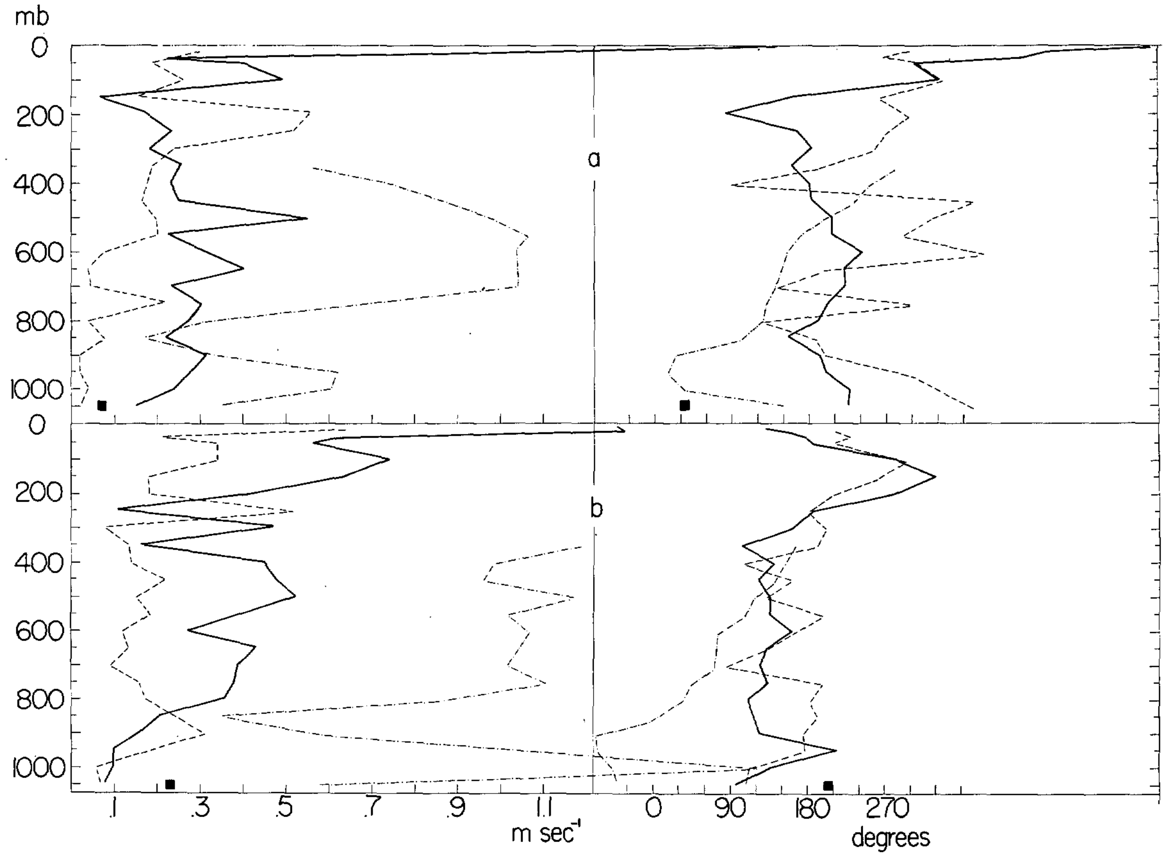

FIGURE 3.—Diurnal variation of (a) eastward and (b) northward wind components in April–July 1956 and 1958 at Eniwetok (solid line); Terceira, Azores (broken lines), after Harris et al. (1962); and Washington, D.C. (dash-dotted lines), after Harris (1959). Filled square denotes data for Bermuda, after Bartrum (1957). Amplitude in m sec^{-1} and phase in degrees counted from 0000 LT.

functions may originate from an arbitrary distribution of forcing. In the case of one set, waves propagate vertically and transport energy away from the level of forcing, phase and energy propagation in general being in the opposite direction. For upward energy propagation, this implies that at a given time the tidal wind vector rotates clockwise with height in the Northern Hemisphere. Energy density is conserved away from levels of forcing and dissipation, so that amplitude increases with height as the square root of the density decreases. Exponential height dependence is characteristic of the other set of Hough functions. The term "trapped" modes is used, since energy does not propagate away from the level of forcing; phase is constant with height.

Harmonic analysis of winds over Eniwetok would seem to support Wallace and Hartranft's findings (1969), based on the wind difference of soundings 12 hr apart, at a large number of upper air stations. More complete observational data, albeit for one station, were at our disposal in the present study, and hodographs of the daily (first plus second harmonics), diurnal and semidiurnal wind variations were plotted for the available 8 sounding hours. Graphs for the various hours were in general agreement, and only the hodographs of 1048 LT (0000 GMT) are reproduced in figure 5 for illustration.

The somewhat irregular pattern of the hodographs for the layer surface to 300 mb would seem compatible with Wallace and Hartranft's suggestion (1969) of a prevalence of trapped and propagating modes in the lower layers.

For the upper troposphere and stratosphere, figure 5 indicates a clockwise turning of the tidal wind vector with height. Amplitude increases with height, although perhaps not exactly at the rate at which the square root of density decreases. This is consistent with Wallace and Hartranft's finding (1969) that vertically propagating modes predominate from about 200 mb upward in low latitudes.

7. RELATION OF WIND AND PRESSURE VARIATIONS

For an analysis of periodic daily variations of wind and pressure, the linearized equations of motion in the following form are appropriate (for example, Wilkes 1949, Harris 1959, Harris et al. 1962):

$$\frac{\partial u}{\partial t}-2\omega \sin \phi v+\frac{1}{\rho_0 a \cos \phi}\frac{\partial p}{\partial \theta}=0 \tag{2}$$

and

$$\frac{\partial v}{\partial t}+2\omega \sin \phi u+\frac{1}{\rho_0 a}\frac{\partial p}{\partial \phi}=0. \tag{3}$$

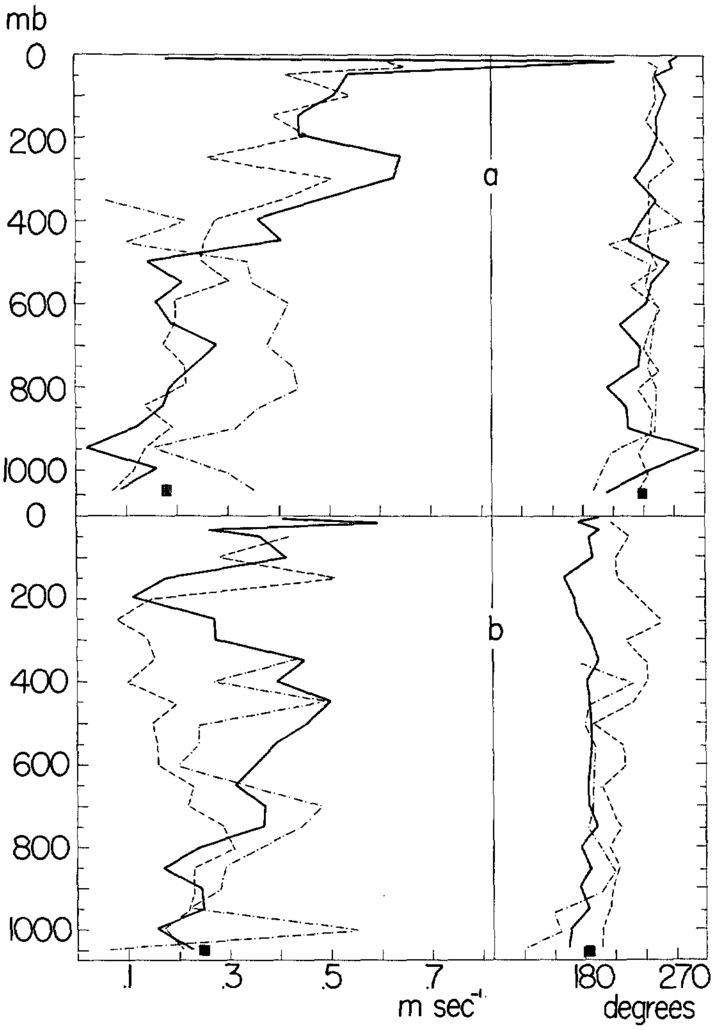

FIGURE 4.—Semidiurnal variation of (a) eastward and (b) northward wind components in April–July 1956 and 1958 at Eniwetok (solid lines); Terceira, Azores (broken lines), after Harris et al. (1962); and Washington, D.C. (dash-dotted lines), after Harris (1959). Filled square denotes data for Bermuda, after Bartrum (1957). Amplitude in m sec^{-1} and phase in degrees counted from 0000 LT.

Here, p denotes the disturbed pressure, ρ_0 the undisturbed density, and u and v the eastward and northward components of the perturbation velocity, respectively; ϕ means latitude, θ longitude, and t time; a and ω are the radius and angular velocity of the earth. Choosing a time factor $e^{\nu i\omega t}$, where ν is the number of the harmonic and $i=\sqrt{-1}$, equations (2) and (3) can be solved for u and v (Wilkes 1949):

$$u=\frac{1}{a\rho_0\omega(4\sin^2\phi-\nu^2)}\left[-\frac{i\nu}{\cos\phi}\frac{\partial p}{\partial\theta}-2\sin\phi\frac{\partial p}{\partial\phi}\right] \quad (4)$$

and

$$v=\frac{1}{a\rho_0\omega(4\sin^2\phi-\nu^2)}\left[-i\nu\frac{\partial p}{\partial\phi}+2\tan\phi\frac{\partial p}{\partial\theta}\right]. \quad (5)$$

The coefficient of the bracketed terms on the right-hand side of equations (4) and (5) is always negative for the semidiurnal wave ($\nu=2$); for the diurnal wave ($\nu=1$), it is negative at $\phi<30°$ and positive poleward of that

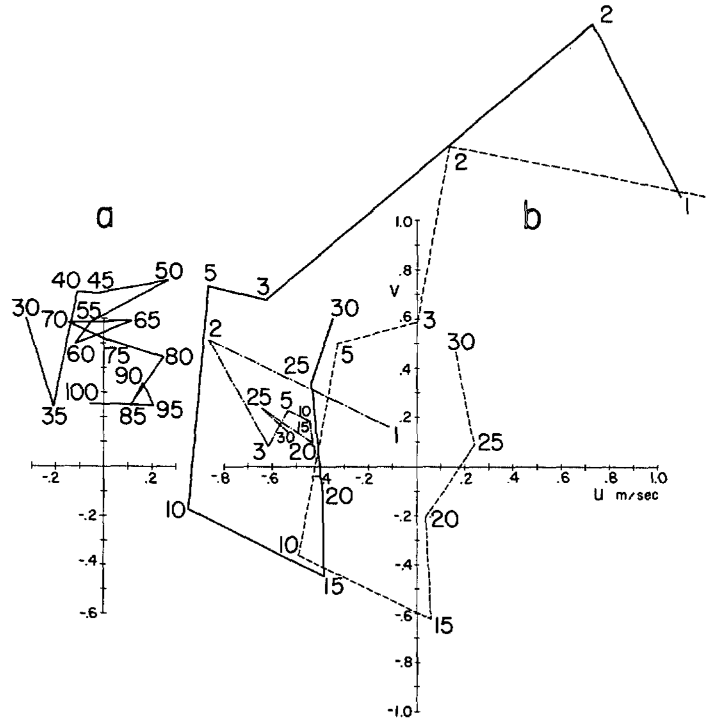

FIGURE 5.—Hodographs of daily (first plus second harmonics) (solid line), diurnal (dash-dotted line), and semidiurnal (broken line) wind variation at 0000 GMT, 1048 LT, during April–July 1956 and 1958 over Eniwetok, surface to 300 mb (a) and 300 to 10 mb (b). Numbers denote end points of vectors and indicate constant pressure levels in decibars.

TABLE 6.—*Observed phase differences in degrees between the first and second harmonics of pressure, p_1, p_2, and eastward and northward wind components, u_1, u_2, v_1, v_2, and theoretical value from equations (4) and (5), $\Delta\alpha$*

Level (mb)	u_1-v_1	u_2-v_2	p_2-u_2	p_2-v_2	p_1-u_1	p_1-v_1
1000	89	56	115	81	243	26
950	12	67	66	89	274	82
900	111	49	52	91	289	2
850	41	36	47	81	28	13
800	98	26	155	91	302	24
750	109	44	121	75	306	17
700	84	54	120	84	289	25
650	90	35	138	83	54	145
600	82	61	113	84	70	12
550	109	60	109	77	40	150
500	107	82	91	83	23	130
450	117	42	131	83	6	112
400	134	56	120	88	14	125
350	122	60	105	75	38	84
300	23	41	129	80	17	40
250	17	68	111	88	35	18
200	162	70	105	85	238	76
150	167	59	106	73	42	125
100	56	63	101	74	140	84
50	118	65	109	84	118	0
30	73	75	90	75	256	3
20	53	83	99	96	294	167
10	89	82	75	67	40	129
$\Delta\alpha$	90	45	90	45	180	90

latitude. For pressure waves progressing uniformly around the earth, with amplitudes decreasing poleward, it is found from equations (4) and (5) that the semidiurnal variation u_2 is $180/\nu$ degrees out of phase with the semidiurnal pressure wave, and for the latitude of Eniwetok this holds also for the diurnal variations u_1 and p_1. Similiarly, the corresponding variations of the northward components v_2 and v_1 lag the respective pressure waves by $90/\nu$ degrees.

Differences between the observed and theoretical phases of the wind and pressure variations (tables 2 and 5 and figs. 1, 3, and 4) are shown in table 6. The quadrature of u and v wind components is only approximately fulfilled: observed phase differences tend to be larger than the theoretical value from equations (4) and (5). Even larger discrepancy will be noticed, however, in relation to the phase of pressure.

With $\partial/\partial t = \omega\partial/\partial\theta$, equation (2) can be written (for example, Harris et al. 1962) as

$$\frac{\partial u}{\partial t}=2\omega \sin \phi v+\frac{g}{a\omega \cos \phi}\frac{\partial z}{\partial t} \tag{6}$$

where g is acceleration of gravity, and z the height variation of a constant pressure surface.

Integrating equation (6) with respect to θ and substituting for v from equation (1) yields

$$z=\frac{2a\omega \sin \phi \cos \phi}{g\nu} C_{v,\nu} \sin\left(\nu\theta-\frac{\alpha_\nu}{\nu}\right)-\frac{a\omega \cos \phi}{g}u \tag{7}$$

where C is the amplitude.

From equation (7), the following relationships between the coefficients P and Q of the cosine and sine terms of the Fourier series can be derived as

$$P_{z,\nu}=-\frac{a\omega \cos \phi}{g}P_{u,\nu}-\frac{2a\omega \sin \phi \cos \phi}{g\nu}Q_{v,\nu}$$

and (8)

$$Q_{z,\nu}=-\frac{a\omega \cos \phi}{g}Q_{u,\nu}+\frac{2a\omega \sin \phi \cos \phi}{g\nu}P_{v,\nu}.$$

Subscripts z, u, v, refer to height and eastward and northward wind components, respectively. At the latitude of Eniwetok, the coefficient of the first terms is approximately 47 sec, and that of the second about 18 sec/ν.

Height variations were computed from those of wind, according to equation (8). Results are included in table 2 for comparison with the analysis of the height data themselves.

Major discrepancies between observed and computed diurnal pressure variations are apparent from table 2. Internal consistency in the phase relationships of the observed u and v variations, however, seems to be somewhat more satisfactory (see table 6).

Harris et al. (1962) have discussed the possibility of a fictitious daily variation of wind resulting from the vertical wind shear and the daily variation of height. From an order of magnitude comparison, they demonstrated that this effect can be safely disregarded. However, these authors emphasized the role of systematic errors in temperature, and hence pressure measurements at stratospheric levels.

8. CONCLUDING REMARKS

Empirical study of atmospheric tidal oscillations has traditionally been hampered by the lack of adequate upper air soundings taken at sufficiently close time intervals. Combination of observation periods with different sounding schedule was thus found to be a useful procedure in earlier studies. The 1956 and 1958 nuclear tests in particular have provided a valuable data collection for such an analysis for various stations in the Marshall Islands area. Characteristics of tidal variations in the troposphere and stratosphere over Eniwetok presented here appear especially interesting in perspective with results available in the literature for a few other locations. Data evaluation for various other stations in the Marshall Islands is presently under way.

ACKNOWLEDGMENTS

This study was supported through National Science Foundation Grant GA-1010. Discussions with D. Watts, Department of Statistics, The University of Wisconsin, are gratefully acknowledged. P. Guetter assisted in the programming for the CDC 3600 and the Univac 1108 Computers.

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[Received September 4, 1969; revised November 20, 1969]